Word problems and strategies for their solutions. (Book: TaL through "Critical Reflective Associations" ...)
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Among the most important types of word problems are those involving common multiples, systems of equations and single and combined rates. Yet, they are all solved essentially by following the same methodology to uncover the link between the problem and its solution:

5 + 1 general steps to solve word problems:
1) **read and analyze the questions** (before the problem’s narrative) paying attention to their essential aspects;
2) **carefully read the problem** to find the relationship between the given data and those sought for, by:
2.1) analyzing **keywords** and **relevant phrases**,
2.2) noticing **irrelevant aspects**, and
2.3) minding "**red herrings**" included to confuse and divert attention from the basic issues at hand;
3) **set up the problem** (making intermediate calculations and conversions if necessary);
4) **translate the verbal statements to algebraic relations**, in order to clarify:
4.1) the mathematically tractable **meaning of the statements**, determining
4.2) the **type of problem**,
4.3) the **dimensional analysis** in the interrelation among the data and operations, and **crucially**
4.4) **what is common, invariant among the variable aspects of the problem**;
5) **resolution and verification**: once the problem is reduced to its essential algebraic form based on §4, you solve it, making sure that each of the steps are correct. There are verbal problems which solutions require for those steps to be partially of entirely repeated. Those techniques can be useful with any kind of multiple choice questions.
6) **verification**: check if your answer is right by checking if its value range makes sense and by quickly going over all operations in reverse. If, say, you are able to save $450 per week and they are asking for how many weeks do you have to work in order to save a total of $1,000 while being paid biweekly, you know the right answer is 6; and, if your exact calculation to a problem in which they are asking you how many children could be fed in a birthday is "16.4", you know the right answer is 16 because you can't feed (2/5) of a child.

Example: A skilled painter approximately needs eight hours to scrape, level and paint 3,600 square meters (m^2). The journal for painting the 5,775m^2 of the interior and exterior walls of a house pays $3,000. He knows that his apprentice is able to finish 150m^2 in half an hour. Would they earn their journal if they can start working at 8AM and must be done by 5PM and his apprentice cannot join him until after 10AM? How much money will each earn? Could one of them earn all the money for himself? Approximate calculations to the dime; that is, round the pennies.

§1: let’s read and analyze each of the questions:

§1.1: Would they earn their journal (i) if they (ii) can start working at 8AM and must be done by 5PM (iii) and his apprentice may not join him until after 10AM? (iv)
i) their journal: they must finish the job in a work day
ii) they: more than one person (two in total) working together (which makes it a combined rate problem)
iii) they start working at 8AM must be done by 5PM: they can maximally work for 9 hours
iv) his apprentice may not join his master until after 10AM: apprentice is not sharing efforts for two hours

§1.2: How much money (v) will each earn (vi)?
v) How much money: certain dollar amount ...
vi) will each earn: more than one person working and sharing (probably not equally) the money (consistent with §ii)

§1.3: Could (only) one of them earn the whole amount of money for himself?
vii) (only) one of them: by himself (that is, do the all work by himself), therefore gain
viii) all the money.

§2: then, carefully read the problem’s story line, parsing its relevant phrases in relation to the questions: A skilled painter approximately needs eight hours to scrape, level and paint 3,600 square meters (m^2). The journal for painting the 5,775m^2 of interior and exterior walls of a house pays $3,000. He knows that his apprentice is able to finish 150m^2 in half an hour.

* **s**killed **p**ainter(ix): one of the painters specified in §ii. Using "Sp" as a suggestive abbreviation needs eight hours(x) to paint 3,600m^2(xi)
* the **t**otal **a**rea (TA) to be painted is 5,775m^2(xii) and the total money (TM) paid as part of the journal is $3,000(xiii)
* his apprentice(xiv), "Ap", does; 150m^2(xv); in half an hour(xvi)

It is helpful, as you learn a type of problem, to mentally recreate the situation or a similar one more fitting to your day-to-day activities. Note that the walls being painted "inside and outside" and that they will have to "scrape, level and paint" is irrelevant, as it would be, say, the names of the painters. The only idea that smells like a red herring is that one of them arrives at 8AM and the other at 10AM. There is nothing specific about the times themselves, just the 2 hour difference the skilled painter had to work by himself. If, say, the problem had outlined the fishy aspect that the skilled painter works with a pneumatic gun, that may be confusing because it messes with the issue of "fairness", "commensurate pay". Yet, **all word problems must be solved using the specified data only**.

§3: Set up of the problem: Tabulation of the basic data.

Painter	Painted Area (m^2)	time (h)	Rate (m^2/h)
Sp (**s**killed)	3,600	8	450
Ap (**a**pprentice)	150	(1/2)	300

§4: Let's translate the logic of the problem to algebraic equations (or inequalities): We know that the **t**otal **a**rea (TA) to be painted the **t**otal **m**oney (TM) are: TA = 5,775m^2 and TM = $3,000.

TA = AS + AA	5,775m^2	*sum of the areas that the skilled painter and the apprentice did*
TM = MS + MA	$3,000	*sum of **m**oney the **s**killed painter and his **a**pprentice each gets*

Now, the skilled painter initially worked two hours by himself. Therefore, he does not have to share the money corresponding to that time interval: How much money should be rightfully his?

(MS(solo)/TM) = (AS(solo)/TA)	*Initially, the amount of money the master makes for himself must correspond with the area he painted by himself*
MS(solo) = (AS(solo)/TA) • TM	*Isolating MS, considering that TM was dividing on the left side of the equation*
MS(solo) = (900m^2 / 5,775m^2)•$3,000 MS(solo) = $467.53...= $467.50	*substituting: Sp paints 900m^2 in two hours and rounding the pennies*

From now on, the remaining area for painting and money to be shared between the two (**2**) are:

A2T = **A2S** + **A2A** = **TA** – AS(solo) **A2T** = 5,775m^2 – 900m^2 = **4,875m^2**	*The total shared area among the two of them is the sum of the area who painted while working together.*
M2T = **M2S** + **M2A** = **TM** – DS(solo) **M2T** = $3,000 – $467.50 = **$2,532.50**	*The total shared money.*

A2S = tS ·450(m^2/h) **A2A** = tA ·300(m^2/h)	*Multiplying the time painting by their respective rate you determine the painted area.*
tS = tA = t2	*Since they share the work, the time they would be working will be the same.*
A2T = **A2S + A2A** **A2T = t2 ·(450(m^2/h) + 300(m^2/h))** **A2T = t2 ·750(m^2/h) = 4,875m^2**	*The timeshare is a common factor. Replacing and adding the like terms (how fast they paint).*
t2 = 4,875m^2 ÷ 750(m^2/h) t2 = 4,875m^2 **x** (1/750)(h/m^2) t2 = 6.5h = 6(5/10)h = 6(1/2)h	*Isolating the time shared and calculating (notice that the dimensions must also be accordingly reversed when a fraction is divided) you come up with approximately 6 hours and 30 minutes.*

... and since the skilled painter had initially worked for two hours by himself, they, as a team, needed 8 hours and 30 minutes, so, they would collect their journal.

Individual work:
1) Using the logic of the steps above, you should then finish the calculations to answer the other two questions.
2) Of all verbal statements translated into algebraic expressions, which one gives off the type of word problem as a combined-rates kind?
3) Create a similar problem (including red herrings) and then team up with one of your colleagues to solve each other's problem methodically following the indicated steps (including the rationalizing T-tables). Teacher will check, both, the problem proposed by each student as well as the blue-pen corrections and solution to the problem of the other student one pairs up with.

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